Steady – State Heat Conduction in Rectangular and Cylindrical Geometries: A Review

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Abstract

This review article aims to consolidate knowledge on steady-state heat conduction by providing a comprehensive overview of the fundamental principles and equations applicable to both rectangular and cylindrical geometries. It evaluates various analytical, numerical, and empirical methods for solving heat conduction problems, highlighting differences in thermal resistance and heat transfer coefficients between the two shapes. The article reviews existing research to pinpoint trends and gaps, discusses practical engineering applications such as thermal management and building materials, and suggests areas for future research, especially in computational techniques and innovative materials. Emphasizing the significance of steady-state conditions in heat transfer systems for their predictability and management, the article underlines the features of steady-state conduction, including homogeneous material properties, equilibrium state, and absence of energy storage, noting its crucial role in enhancing the performance and energy efficiency of engineering systems.

Keywords: Historical Background; Rectangular Co-Ordinate; Cylindrical Co-Ordinate; Steady One-Dimensional Conduction; Heat Generation; Rectangular Slab; Solid Wire; Hollow Wire.

1. Introduction and Historical Background

1.1 Introduction

Heat transfer in thermodynamic systems refers to the movement of heat across a system's boundaries due to a temperature difference between the system and its environment.

Steady-state conduction differs from transient heat transfer in that it involves a constant rate of heat transfer throughout the material. In steady-state conduction, the temperature remains uniform over time. This mode of conduction occurs when the temperature gradient driving the process is stable, resulting in a spatial temperature distribution that does not change after an initial equilibration period. Consequently, while the temperature can vary at different locations within the object, any fluctuations over time will remain constant. In this state, the amount of heat entering any part of the object equals the amount of heat exiting it; otherwise, the temperature in that region would either rise or fall due to excess thermal energy accumulation.

For instance, imagine a bar with one end cold and the other hot. Once steady-state conduction is established, the temperature gradient along the bar becomes static over time, with each cross-section perpendicular to the heat transfer direction maintaining a constant temperature. In the absence of heat generation within the rod, this temperature will change linearly along its length.

In steady-state conduction, similar principles apply to heat currents as those in direct current electrical conduction. Thermal resistances can be viewed as analogs to electrical resistances, with temperature corresponding to voltage and the rate of heat transfer (heat power) equivalent to electric current. Steady-state systems can be effectively modeled using networks of thermal resistances arranged in series and parallel, mirroring electrical resistor networks.

When conducting a Steady State Heat Transfer Analysis, the objective is to assess the thermal conditions of a system in equilibrium. This process requires a solid understanding of the underlying physics and technical proficiency in numerical methods. To streamline this complex procedure, follow these steps:

1. Identify the system: Start by specifying the system you wish to analyze—whether it be a heat exchanger, radiator, or a whole building. This clarity will help establish the defined boundaries for the heat transfer process.
2. Understand the physical model: Develop a mathematical representation of the physical processes at play. This includes identifying key variables such as the material's thermal conductivity, the areas involved in heat transfer, and the temperature gradient. For conduction analysis, apply Fourier's Law.
3. Establish boundary conditions: Define the constraints of the system, including specified temperatures or heat fluxes at certain boundaries, as these will influence the solution's determinacy. Accurate boundary conditions are crucial since they dictate how heat transfers from the system's edges to its internal points.
4. Solve the mathematical model: After establishing the mathematical model and boundary conditions, choose an appropriate numerical method to solve it. The resulting solutions will yield the temperature distribution within the system.
5. Perform error checks: Verify the model's accuracy through various means, such as ensuring energy conservation in steady-state conditions, comparing results with available analytical solutions, or refining numerical simulations with a finer mesh.
6. Analyze results: Finally, evaluate the solutions obtained. This step may involve calculating heat flux, identifying regions with elevated temperatures, and assessing the effectiveness of

insulation or cooling methods. The insights garnered from this analysis will inform practical modifications or designs for improved thermal management [1] – [3].

The present review article aims to consolidate and analyze knowledge within the field of heat conduction through the following: Offering a detailed overview of the governing principles and equations for steady-state heat conduction in both rectangular and cylindrical shapes; Assessing various techniques i.e. analytical, numerical, and empirical for solving heat conduction problems in these geometries; Highlighting the differences in heat conduction behavior between rectangular and cylindrical geometries, focusing on thermal resistance and heat transfer coefficients; Reviewing existing studies to identify trends and gaps in current research on steady-state heat conduction; Discussing practical applications in engineering and manufacturing, including thermal management and building materials; Suggesting areas for future exploration, particularly in computational methods and innovative materials; Acting as a foundational guide for students and professionals by summarizing key concepts and applications related to steady-state heat conduction [4], [5] and [6].

1.2 Historical Background

The exploration of steady state heat conduction has transformed from its early philosophical beginnings into a complex field characterized by mathematical precision and practical relevance. In facing challenges related to energy efficiency, climate change, and advancements in materials technology, the principles of steady state heat conduction remain crucial in deepening our understanding of thermal processes and their applications in contemporary engineering and daily life. Steady state heat conduction is a key concept in thermodynamics and heat transfer, referring to the process of thermal energy transfer through a material in which the temperature remains constant over time. This constancy indicates that there is a balance between the heat entering and leaving the system. The origins of heat conduction can be traced back to ancient Greece, where philosophers such as Aristotle speculated about the nature of heat and matter. It wasn't until the Renaissance that more systematic investigations into heat began, driven by advancements in experimental methodologies and scientific exploration.

In the 17th and 18th centuries, the scientific understanding of heat transfer was greatly enhanced through the development of thermodynamics. Pioneering figures like Robert Boyle, Thomas Newcomen, and James Watt contributed to the understanding of heat as a form of energy. The caloric theory, articulated by Antoine Lavoisier and others, proposed that heat was a fluid (caloric) flowing from warmer to cooler objects; this theory prevailed until the 19th century, when new insights began to alter the understanding of heat transfer.

A significant advancement in the study of heat conduction came with Jean-Baptiste Joseph Fourier's formulation of Fourier's Law in his 1822 work entitled Analytical Theory of Heat. This law posits that the rate of heat transfer through a material is directly proportional to the negative temperature gradient and the area across which heat flows. This established the essential mathematical framework for modeling heat conduction, forming the basis for modern heat transfer analysis.

The 19th century saw the emergence of calculus and advanced mathematics, thanks in part to mathematicians like Carl Friedrich Gauss and Augustin-Louis Cauchy. These developments allowed for a more precise mathematical description of heat conduction. The creation of partial differential equations led to the formulation of the heat equation, which characterizes how heat spreads through a material over time.

The principles of steady state heat conduction have found widespread applications in engineering and technology, impacting the design of thermal insulation, heat exchangers, electronic devices, and various industrial processes. Accurate predictions of temperature distribution in steady-state conditions have become essential for optimizing designs and ensuring safety in numerous fields.

Today, steady state heat conduction is analyzed using a variety of tools, including numerical methods and computer simulations. Innovations in materials science have prompted investigations into new materials with distinctive thermal properties, enhancing the efficiency of heat transfer systems [7] – [11].

2. General Conduction Equation for Rectangular and Cylindrical Coordinates

2.1 Rectangular Coordinates

The general conduction equation applicable to a three-dimensional solid body experiencing uniform internal heat generation such as that from ohmic heating (the atomic heating of matter at the molecular level) can be expressed in terms of temperature variations over time. This equation considers both the spatial distribution of temperature within the material and the temporal changes in temperature.

Consider an element at a temperature t, for any instant of time τ, through the solid, homogeneous body shown in Figure 1 below. Let the internal heat generation rate per unit volume be $\dot{q}$ and let the density of the material be ρ, the specific heat capacity C, and the thermal conductivity k; Assume that these properties are regular and constant over time.

Using Fourier's Law of Conduction, which says (the rate of heat flow through a single solid, homogeneous metal is directly proportional to the cross-sectional area perpendicular to the direction of flow and to the change in temperature with respect to the length of the flow path, $\frac{dt}{dx}$ (this is an experimental law based on observation).

$$Q\alpha - A\frac{dt}{dx}$$

$$The\ flow\ is\ in\ the\ x\ direction, Q = -kA\frac{dt}{dx}$$

$$Qdx = -kAdt$$

$$\int_0^x Qdx = -\int_{t_1}^{t_2} kAdt$$

$$Qx = -kA(t_2 - t_1)$$

$$\therefore Q = \frac{-kA}{x}(t_2 - t_1) or Q = \frac{kA}{x}(t_1 - t_2)$$

$$Q_x = -kA\frac{\partial t}{\partial x} = -k(dydz)\frac{\partial t}{\partial x}$$

$$Q_y = -kA\frac{\partial t}{\partial y} = -k(dxdz)\frac{\partial t}{\partial y}$$

$$Q_z = -kA\frac{\partial t}{\partial z} = -k(dxdy)\frac{\partial t}{\partial z}$$

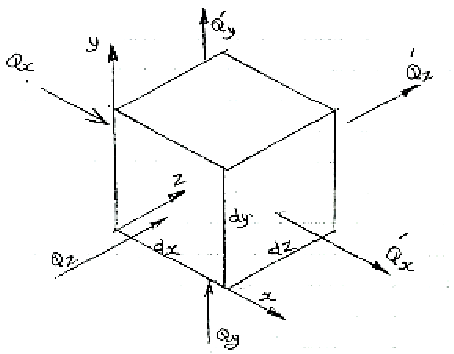

Figure 1: A Small Element of a Homogeneous Solid Body

The change in heat flow in the x direction,

$$Q'_x - Q_x = \frac{\partial Q}{\partial x} dx = -k \frac{\partial^2 t}{\partial x^2} dxdydz$$

The same applies to heat flow in the y and z directions.

$$Q'_y - Q_y = \frac{\partial Q}{\partial y} dy = -k \frac{\partial^2 t}{\partial y^2} dxdydz$$

$$Q'_z - Q_z = \frac{\partial Q}{\partial z} dz = -k \frac{\partial^2 t}{\partial z^2} dxdydz$$

$$rate\ of\ heat\ generation,\ Q = \dot{q}(dxdydz)$$

$$Rate\ of\ energy\ increase\ for\ an\ element$$
$$=\ mass\ x\ specific\ heat\ x\ rate\ of\ change\ in\ temperature\ with\ respect\ to\ time$$

$$\therefore Rate\ of\ energy\ increase\ for\ an\ element\ = \rho(dxdydz)C \frac{\partial t}{\partial \tau}$$

The energy balance of the element is given by the following equation:

$$Rate\ of\ energy\ increase\ of\ element\ =\ rate\ of\ heat\ generation\ -\ change\ in\ heat\ flow$$

$$\dot{q}(dxdydz) - \left[(Q'_x - Q_x) + \left(Q'_y - Q_y\right) + (Q'_z - Q_z)\right] = \rho C(dxdydz) \frac{\partial t}{\partial \tau}$$

It can be expressed as follows:

$$\dot{q}(dxdydz) - \left[-k \frac{\partial^2 t}{\partial x^2} dxdydz - k \frac{\partial^2 t}{\partial y^2} dxdydz - k \frac{\partial^2 t}{\partial z^2} dxdydz\right]$$
$$= \rho C(dxdydz) \frac{\partial t}{\partial \tau}$$

By dividing both sides of the equation by $(dxdydz)$, we get:

$$\dot{q} - \left[-k \frac{\partial^2 t}{\partial x^2} - k \frac{\partial^2 t}{\partial y^2} - k \frac{\partial^2 t}{\partial z^2}\right] = \rho C \frac{\partial t}{\partial \tau}$$

By dividing both sides of the equation by k, we obtain:

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho C}{k} \frac{\partial t}{\partial \tau}$$

But $\frac{k}{\rho C} = \alpha$ (thermal diffusivity).

Thermal diffusivity is the ratio between thermal conductivity k and heat capacity ρC.

If the value of α is large, it means either a large value of k or a small value of ρC. In the first case, there is rapid heat transfer, and in the second case, the absorption of heat by the body is small.

Thus, the above equation can be written as follows:

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial t}{\partial \tau}\ (unstable\ three - dimensional\ equation)$$

If the equation is stable in three dimensions, then $\frac{\partial t}{\partial \tau} = 0$, it can therefore be expressed as follows:

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{\dot{q}}{k} = 0$$

If the system is stable in two dimensions,

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\dot{q}}{k} = 0$$

If the system is stable in one dimension,

$$\frac{\partial^2 t}{\partial x^2} + \frac{\dot{q}}{k} = 0$$

Refer to references [4], [12] and [13].

2.2 Cylindrical Coordinates

Consider the flow of heat through a small ring element of thickness dr at any radius r, where the temperature is t. Let the thermal conductivity of the material be k.
For a unit of length in the axial direction, as in Figure 2 below, the energy balance equation can be written as follows:

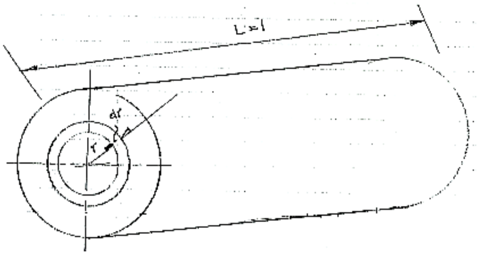

Figure 2: A Small Cylindrical Element of a Solid Body
The energy balance equation for the element,

$$\dot{q}2\pi r dr - \frac{\partial Q}{\partial r}dr = \rho C 2\pi r \frac{\partial t}{\partial \tau}$$

$$\dot{q}2\pi r dr - \frac{\partial}{\partial r}\left[-k2\pi r \frac{\partial t}{\partial r}\right]dr = \rho C 2\pi r dr \frac{\partial t}{\partial \tau}$$

By dividing both sides of the equation by $2\pi dr$:

$$\dot{q}r + \frac{\partial}{\partial r}\left(kr\frac{\partial t}{\partial r}\right) = \rho C r \frac{\partial t}{\partial \tau}$$

$$\therefore \dot{q}r + \left[kr\frac{\partial^2 t}{\partial r^2} + k\frac{\partial t}{\partial r}\right] = \rho C r \frac{\partial t}{\partial \tau}$$

By dividing the numerator and denominator by kr:

$$\therefore \frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

Knowing the temperature distribution throughout a given body is of great importance in many engineering problems. This information will be useful in calculating heat gained and heat lost in the body. It is useful in designing boilers, turbines, jet engines, and casting and molding dies [4] and [14].

3. Stable One-Dimensional Conduction with Heat Generation

3.1 Rectangular Slab

Example: Consider a wall of width L, one side of which is insulated as shown in Figure 3 below. Let the temperature of the free face be T_1, and keep the value of both $q^{\cdot}$ and k constant. Determine the maximum temperature in the wall.
Where $q^{\cdot}$ = heat generated per unit volume.
k = thermal conductivity.
Solution:

$$\frac{d^2 t}{dx^2} + \frac{\dot{q}}{k} = 0$$

By performing the integration,

$$\frac{dt}{dx} + \dot{q}\frac{x}{k} = C_1 \quad (1)$$

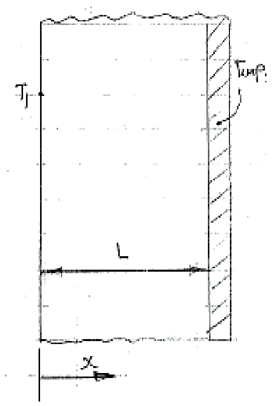

Figure 3: Rectangular Slab

By integrating again,

$$t(x) + \dot{q}\frac{x^2}{2k} = C_1 x + C_2 \quad (2)$$

Applying the boundary conditions (B.C) to obtain the values of C_1 and C_2.

When $x = 0, t(x) = T_1$

By substituting into equation (2),

$$T_1 + 0 = 0 + C_2$$

$$\therefore C_2 = T_1$$

When $x = L, \frac{dt}{dx} = 0$ and by substituting into equation (1),

$$\therefore C_1 = \frac{\dot{q}L}{k}$$

$$\therefore t(x) + \frac{\dot{q}x^2}{2k} = \frac{\dot{q}L}{k}x + T_1$$

$$t(x) - T_1 = \frac{\dot{q}Lx}{k}\left(1 - \frac{x}{2L}\right)$$

$$\therefore t(x) = T_1 + \frac{\dot{q}Lx}{k}\left(1 - \frac{x}{2L}\right)$$

The maximum temperature occurs at x = L,

$$\therefore t(x) = T_1 + \frac{\dot{q}L^2}{2k} \quad (3)$$

3.2 Solid Wire

Consider a solid wire carrying a current of I ampere, as shown in the Figure 4 below.

Conduction equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dt}{dr}\right) + \frac{\dot{q}}{k} = 0$$

Multiply by r, and perform the integration,

$$\frac{d}{dr}\left(r\frac{dt}{dr}\right) + \frac{\dot{q}r}{k} = 0$$

$$r\frac{dt}{dr} + \frac{\dot{q}r^2}{2k} = C_1$$

Divide by r, perform the integration,

$$\frac{dt}{dr} + \frac{\dot{q}r}{2k} = \frac{C_1}{r}$$

$$t(r)+\frac{\dot{q}r^2}{4k}=C_1\,ln\,r+C_2$$

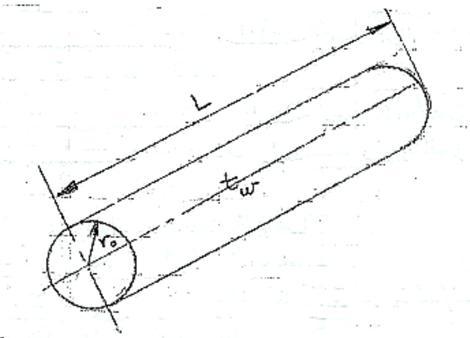

Figure 4: Solid Wire

Boundary Conditions:

When $r\ =\ 0$, there is no heat transfer (the line of symmetry acts as an insulator).

$$dt/_{dr}=0,\therefore C_1=0$$

When $r\ =\ r_0, t(r)=t_w$

$$\therefore t_w+\frac{\dot{q}r_0^2}{4k}=C_2,\therefore C_2=t_w+\frac{\dot{q}r_0^2}{4k}$$

$$t(r)+\frac{\dot{q}r_0^2}{4k}=C_2,\therefore C_2=t_w+\frac{\dot{q}r_0^2}{4k}$$

$$t(r)=t_w+\frac{\dot{q}r_0^2}{4k}-\frac{\dot{q}r^2}{4k}$$

$$t(r)=t_w+\frac{\dot{q}r_0^2}{4k}\left(1-\left(\frac{r}{r_0}\right)^2\right)$$

The maximum temperature $t(max)$ occurs at $r\ =\ 0$

$$t(max)=t_w+\frac{\dot{q}r_0^2}{4k}$$

The rate of heat transfer can be calculated from Fourier's law:

$$q=-kA\frac{dt}{dr}$$

3.3 Hollow Wire

Consider a hollow wire as shown in the Figure 5 below:

Boundary conditions (B. conditions)

When $r\ =\ r_i, t\ =\ t_i,$

And when r = r_0, $t\ =\ t_0,$

By applying the above boundary conditions,

$$t-t_0=\frac{\dot{q}}{4k}(r_0^2+r^2)+C_1\,ln\frac{r}{r_0}$$

Where C_1equals,

$$C_1=\frac{(t_i-t_0)+\dot{q}\,\frac{(r_i^2-r_o^2)}{4k}}{ln\frac{r_i}{r_0}}$$

Refer to references [4], [15] and [16].

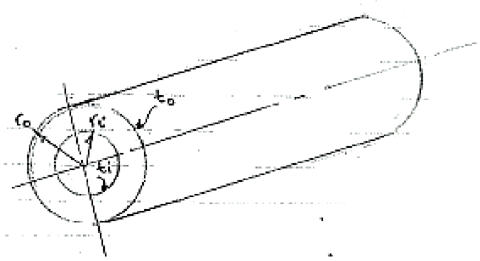

Figure 5: Hollow Wire

4. Differentiating Various Methods for Solving Heat Conduction Equations

Heat conduction is a fundamental phenomenon in thermodynamics and engineering, playing a crucial role in various applications, from building design to electronics cooling. Solving heat conduction problems involves various analytical, numerical, and empirical methods, each with its own strengths and weaknesses.

Solving heat conduction problems involves a blend of analytical, numerical, and empirical methods tailored to the complexity of the system under study. Each approach has its specific advantages, particularly in the context of different shapes. Understanding the nuances of thermal resistance and heat transfer coefficients is essential for optimizing thermal management in engineering applications, leading to improved efficiency and performance in thermal systems.

4.1 Analytical Methods

Analytical methods focus on deriving closed-form solutions to heat conduction problems using mathematical modeling. The most common approach is applying Fourier's law of heat conduction and solving the heat conduction equation under specific boundary and initial conditions. These solutions are predominantly beneficial for simple geometries, such as rectangular slabs, cylinders, or spheres, allowing for accurate predictions of temperature distribution and heat flux.

The main analytical techniques include:

4.1.1 Separation of Variables

This technique is used for linear problems to decompose the governing equations into simpler ordinary and partial differential equations and equations for exponential growth and decay. The point of separation of variables is not just to get some solution, but to get a general solution, which can be used to produce a solution for any initial condition. It is one of the most widely used techniques to solve partial differential equations and is based on the assumption that the solution of the equation is separable, that is, the final solution can be represented as a product of several functions.

4.1.2 Laplace Transform

This method provides solutions in the transformed domain, which can simplify solving partial differential equations. It is accepted widely in many fields. The Laplace transform simplifies a given linear differential equation to an algebraic equation, which can later be solved using the standard algebraic identities. The Laplace equations are used to describe the steady-state conduction heat transfer without any heat sources or sinks.

The Laplace Transform can be used to solve differential equations using a four-step process:

1. Take the Laplace Transform of the differential equation using the derivative property (and, perhaps, others) as necessary.
2. Put initial conditions into the resulting equation.
3. Solve for the output variable.
4. Get result from Laplace Transform tables. If the result is in a form that is not in the tables, you'll need to use the Inverse Laplace Transform.

4.2 Numerical Methods

Numerical methods are essential for solving complex heat conduction problems where analytical solutions are infeasible due to irregular geometries or varying material properties. Some common numerical techniques include:

4.2.1 Finite Difference Method (FDM): This method approximates derivatives by difference equations and is particularly useful in time-dependent problems.

4.2.2 Finite Element Method (FEM): A powerful technique that breaks the problem domain into smaller, simpler parts (elements) to analyze complex geometries and varying properties.

4.2.3 Computational Fluid Dynamics (CFD): Often used in conjunction with heat transfer analysis, CFD can model heat conduction in systems involving fluid flow, providing insights into convective conditions.

4.3 Empirical Methods

Empirical methods rely on experimental data to derive correlations and relationships. These approaches are often used to establish heat transfer coefficients and thermal resistance values, particularly when dealing with real-world materials or complex geometries where theoretical models may fall short. Common applications include:

4.3.1 Heat Exchanger Design: Correlation equations derived from experimental data are used to estimate overall heat transfer coefficients and thermal resistances in heat exchangers.

4.3.2 Thermal Performance Monitoring: Empirical studies can inform assessments of material performance in varied conditions, influencing insulation material choices and thermal barrier designs.

4.4 Thermal Resistance and Heat Transfer Coefficients

When comparing different shapes, the thermal resistance and heat transfer coefficients are key parameters influencing how heat is conducted through materials.

4.4.1 Thermal Resistance (R): is determined by the geometry of the material, its thickness, and conductivity (k). It represents the opposition to heat flow. For simple shapes, thermal resistance can often be calculated using the formula:

$$R = \frac{L}{kA}$$

where (L) is the thickness, (k) is the thermal conductivity, and (A) is the cross-sectional area.

4.4.2 Heat Transfer Coefficient (h): varies significantly between shapes due to the influence of surface conditions, flow characteristics, and the nature of heat transfer (conduction, convection, or radiation). In fluid-solid interfaces, h plays a critical role, with different geometries yielding vastly different transfer efficiencies.

Refer to references [4], and [17] – [20].

5. Conclusion

Heat transfer is a crucial concept in engineering, especially steady-state heat transfer, which refers to a condition where temperature and heat transfer rates remain constant over time, ensuring stable thermal flow .

This review article provides a comprehensive overview of steady-state heat conduction principles and equations for rectangular and cylindrical geometries. It assesses various methods for solving heat conduction problems and examines the differences in behavior between these geometries, focusing on thermal resistance and heat transfer coefficients. The article also highlights trends and gaps in existing research, discusses applications in thermal management and building materials, and proposes future research areas in computational methods and new materials.

Historically, the study of heat conduction has transitioned from philosophical ideas to a mathematically rigorous field, originating in ancient Greece and evolving through key scientific advancements in the Renaissance and 17th to 18th centuries, with contributions from scientists like

Boyle and Lavoisier. Fourier's Law marked a significant development in heat transfer analysis, while the 19th century introduced calculus and partial differential equations, leading to the formulation of the heat equation. Today, steady-state heat conduction is essential in various engineering applications, from thermal insulation to electronics, and benefits from numerical simulations and advancements in materials science to enhance heat transfer efficiency.

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