Comprehensive Insights into Convective Heat Transfer Mechanisms

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Abstract

This paper presents a comprehensive introduction to convective heat transfer, a critical process in various engineering applications and natural phenomena. We begin by defining the fundamental concepts and terminology associated with convective heat transfer, delineating its significance in thermal management and fluid dynamics. Theoretical foundations, including the governing equations and boundary layer concepts, are discussed in detail. A mathematical analysis is performed to derive key relationships and principles governing convective heat transfer, including the role of convective heat transfer coefficients, Nusselt number, and Prandtl number. We also explore different regimes of convection, namely forced and natural convection, and their implications in practical scenarios. Through a combination of analytical methods and case studies, we illustrate the impact of various parameters on convective heat transfer rates. The findings underscore the importance of understanding convective processes for optimizing thermal systems in engineering designs.

Keywords: Forced Convection; Dimensional Analysis; Reynolds's Analogy; Heat Exchanger Effectiveness; Natural Convection.

1. Overview

Convection is a fundamental mode of heat transfer that occurs when a fluid, whether it be a liquid or a gas, undergoes movement due to temperature differences within the fluid or surrounding environment. This movement can be driven by a variety of factors, including the expansion and contraction of fluids, the displacement of denser fluids by less dense ones, and the application of external forces through mechanisms such as pumps or fans.

In natural convection, the primary driving force behind fluid movement is the buoyancy effect, which is a direct result of the difference in density between hot and cold fluids. As a fluid heats up, its density decreases, causing it to rise through the surrounding environment. Conversely, as a fluid cools down, its density increases, causing it to sink. This natural process is responsible for various phenomena, such as the draft in a chimney or around any fire, where the movement of hot fluids creates a circulation of air that helps to dissipate heat.

In addition to natural convection, forced convection also plays a significant role in many engineering applications. Forced convection involves the use of external mechanisms to create a flow of fluid, which can be driven by a variety of factors, including the rotation of a turbine, the movement of a pump, or the use of a fan. This type of convection is commonly seen in applications such as automotive engines, where a water pump creates a flow of coolant that is necessary for the efficient operation of the engine.

The convection heat transfer mode is comprised of two primary mechanisms: diffusion and advection. Diffusion refers to the transfer of energy through the random motion of individual molecules, while advection refers to the transfer of energy through the bulk movement of the fluid. The combination of these two mechanisms results in a cumulative transport of energy, which is a fundamental aspect of convection.

In many real-world applications, natural and forced convection occur simultaneously, a phenomenon known as mixed convection. This can be seen in applications such as solar central receivers or cooling of photovoltaic panels, where a combination of natural convection due to temperature gradients and forced convection due to external mechanisms, such as a fan, work together to enhance heat transfer.

Convection can be further classified based on the type of flow involved. Internal flow refers to the movement of a fluid through a confined space, such as a pipe or a duct, while external flow refers to the movement of a fluid over a solid surface. Both internal and external flow can occur with either natural or forced convection, making it a complex and multifaceted mode of heat transfer.

The classification of convection can also be made based on the smoothness and undulations of the solid surfaces involved. While a majority of the information on convection deals with smooth surfaces, many applications involve wavy or irregular surfaces, such as solar collectors or regenerative heat exchangers. These surfaces can significantly affect the flow and heat transfer characteristics of convection, making it essential to develop mathematical models that account for their presence.

One way to visually demonstrate the principles of natural convection is through the use of a glass container filled with hot water and red food dye. When placed inside a fish tank with cold, clear water, the convection currents of the red liquid can be observed rising and falling in different regions, illustrating the process as heat gradients are dissipated. This experiment provides a simple and effective way to visualize the fundamental principles of convection and its role in heat transfer, [1], [2], [3], [4] and [5].

2. Forced Convection

The study of forced convection is related to the transfer of heat between a moving fluid and a solid surface. In order to apply Newton's law of cooling given by equation$\left(Q = hA(t_w - t)\right)$, it is necessary to find a value for the heat transfer coefficient, h. It has been mentioned that h is given by k/δ; Where k is the thermal conductivity of the fluid and δ is the thickness of the fluid film on the surface. Therefore, the problem is to find a value for δ in terms of the fluid properties and fluid velocity. The thickness of the fluid film or slice δ depends on the type of fluid flow over the surface and this is governed by the Reynolds number, Re. Below, Figure 1 illustrates the mechanism of heat transfer via forced convection.

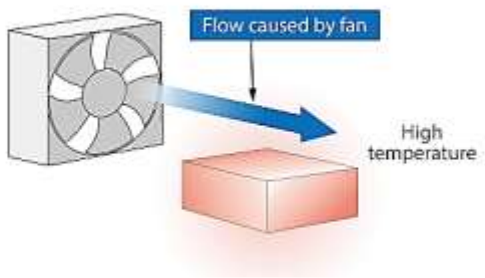

Figure 1: Forced Convection Heat Transfer Method

The Reynolds number is a dimensionless group given by:

$$Re = \frac{\rho C L}{\mu} \; or \; \frac{CL}{\nu}$$

(Where, ρ = density of the fluid; C = average velocity of the fluid; L = characteristic linear dimension); μ = dynamic viscosity of the fluid; ν = kinematic viscosity of the fluid, μ/ρ).

Various forms of forced convection such as flow in a pipe, flow through a pipe, and flow across a flat plate can be addressed mathematically by making specific assumptions about boundary conditions. While deriving an exact mathematical solution for these scenarios is quite challenging, particularly in cases involving turbulent flow, approximate solutions can be achieved by employing suitable assumptions.

However, it is important to note that many heat transfer results are derived from experimental tests, and for numerous problems, no mathematical solutions exist; instead, experimental data takes precedence. This empirical data can be generalized using proper dimensional analysis [6] – [13].

3. Dimensional Analysis

In order to apply dimensional analysis, it is necessary to know all the variables on which the required or desired function depends from experiment or experience. The results must be applied to geometrically similar objects, so one of the variables must always have a characteristic linear dimension.

Consider a dimensional analysis of forced convection, assuming that free convection effects due to density differences are ignored. It was found that the heat transfer coefficient h depends on the viscosity of the fluid μ, the density of the fluid ρ, the thermal conductivity k, the specific heat of the fluid c, the temperature difference between the surface and the fluid θ, and the speed of the fluid C. Thus, we get:

$$h = f(\mu, P, k, c, \theta, C, L) \quad (1)$$

(where L is a characteristic linear dimension, and f is a function)

Equation (1) can be written as follows,

$$h = A \mu^{a_1} \rho^{b_1} k^{c_1} c^{d_1} \theta^{e_1} C^{f_1} L^{g_1} + B \mu^{a_2} \rho^{b_2} k^{c_2} c^{d_2} \theta^{e_2} C^{f_2} L^{g_2} + etc \quad (2)$$

(Where A and B are constants, and a_1, b_1, $c_1, d_1, etc.$ are arbitrary indices)

Each element on the right side of the equation must have the same h-dimensions. Considering the first element only, the following can be written:

$$h \; dimension = \mu^{a_1} \rho^{b_1} k^{c_1} c^{d_1} \theta^{e_1} C^{f_1} L^{g_1}$$

Each of the properties in the above equation can be expressed in terms of the five basic dimensions: $mass\; M, length\; L, time\; T, temperature\; t, and\; heat\; Q.$

$For\; h\; the\; units\; are\; \frac{W}{m^2 K}\; ;\; and\; the\; dimensions\; are\; \frac{Q}{L^2 T t}$

$For\; \mu\; the\; units\; are\; \frac{kg}{ms}\; ;\; and\; the\; dimensions\; are\; \frac{M}{LT}$

$For\ k\ the\ units\ are\ \dfrac{W}{mK}\ ;\ and\ the\ dimensions\ are\ \dfrac{Q}{LTt}$
$For\ \rho\ \ the\ units\ are\ \dfrac{kg}{m^3}\ ;\ and\ the\ dimensions\ are\ \dfrac{M}{L^3}$
$For\ c\ \ the\ units\ are\ \dfrac{kj}{kgK}\ ;\ and\ the\ dimensions\ are\ \dfrac{Q}{Mt}$
$For\ L\ \ \ the\ units\ are\ \ m\ ;\ and\ the\ dimensions\ are\ L$
Therefore, by compensation:

$$\frac{Q}{L^2Tt} = \left(\frac{M}{LT}\right)^a \left(\frac{M}{L^3}\right)^b \left(\frac{Q}{LTt}\right)^c \left(\frac{Q}{MT}\right)^d (t)^e \left(\frac{L}{T}\right)^f (L)^g$$

By grouping similar terms,

$$\frac{Q}{L^2Tt} = (\mu)^{a+b-d}(L)^{f+g-a-3b-c}(T)^{-a-c-f}(t)^{e-c-d}(Q)^{c+d}$$

For the dimensions of both sides of an equation to be the same, the exponent of each fundamental dimension must be the same on both sides of the equation.
Therefore, by equalizing the exponents on both sides of the equation, we get:
)i (: $1 = c + d$ for Q
)ii (: $-2 = f + g - a - 3b - c$ for L
)iii (: $-1 = -a - c - f$ for T
)iv (: $-1 = e - c - d$ for t
)v (: $0 = a + b - d$ for M
We now have five equations and seven unknown exponents; Therefore, a solution can only be obtained in terms of the semantics of two of the exponents. a, b, c, e and g are best expressed in terms of d and f. Therefore, it can be explained that:

$$a = (d - f);\ b = f;\ c = (1 - d);\ e = 0;\ g = (f - 1)$$

Substituting these values into equation (2), we get:

$$h = A\,\mu^{(d_1-f_1)}\rho^{b_1}k^{(1-d_1)}c^{d_1}\theta^{0}C^{f_1}L^{(f_1-1)} + B\,\mu^{a_2}\rho^{(d_2-f_2)}k^{(1-d_2)}c^{d_2}\theta^{e_2}C^{f_2}L^{(f_2-1)} + etc$$

$$i.e.\ h = A\frac{k}{L}\left(\frac{c\mu}{k}\right)^{d_1}\left(\frac{\rho CL}{\mu}\right)^{f_1} + B\frac{k}{L}\left(\frac{c\mu}{k}\right)^{d_2}\left(\frac{\rho CL}{\mu}\right)^{f_2} + etc$$

Therefore, it can be observed that,

$$\frac{hL}{k} = KF\left\{\left(\frac{c\mu}{k}\right) ؛ \left(\frac{\rho CL}{\mu}\right)\right\}$$

(Where K is a constant and F is a function). The dimensionless group, hL/k, is called the Nusselt number Nu; The dimensionless group, $c\mu/k$, is called the Prandtl number, Pr; The dimensionless group, $\rho CL/\mu$, is the Reynolds number Re.

$$Nu = KF\{Pr, Re\} \quad (3)$$

Experiments are conducted to calculate K and determine the nature of the function F.
When evaluating $Nu, Pr, and\ Re$, it is necessary to consider the properties of the fluid at an appropriate average temperature, since the properties change with temperature change. For cases where the temperature of most of the fluid is not significantly different from the temperature of the solid surface, therefore, the properties of the fluid are evaluated at an average temperature (i.e. Mean bulk temperature).
When the temperature difference is large, errors arise due to using the average temperature of most of the fluid. To solve this problem, the mean film temperature is sometimes used, which is defined as:

$$t_f = \frac{t_b + t_w}{2} \quad (4)$$

(where t_b is the temperature of the bulk of the fluid, and t_w is the surface temperature)

When using an empirical equation, it is important to know at what reference temperature the properties are being evaluated by the person performing the test. It should be noted that the Prandtl number, $Pr = c\mu/k$; It is all composed of properties of a fluid and is itself a property.
For laminar flow in a pipe an exact mathematical solution is found, this gives $Nu = 3.65$. It can be seen that, since $Nu = hd/k = 3.65$; The heat transfer coefficient, h, for any pipe depends only on the thermal conductivity of the fluid.
In the previous dimensional analysis, five basic dimensions were chosen: $temperature\ Q, length\ L, time\ T, temperature\ t, and\ mass\ M.$
Units of work or energy are generally given by:

$$(Acceleration \times mass \times distance) = (force \times distance) = energy$$

$$FS = maS$$

$$= M\frac{L}{T^2}L = \frac{ML^2}{T^2}$$

Since heat is a form of energy and is a derivative dimension of the fundamental dimensions, it can be seen that there is no need to choose heat as one of the fundamental dimensions. If Q is eliminated, and the temperature dimensions are replaced by $\frac{ML^2}{T^2}$, then four dimensionless groups are obtained, whenever this happens.

$$Nu = KF\left\{Pr, Re, \frac{C^2}{c\theta}\right\}$$

Now, if the group $\frac{C^2}{c\theta}$ is divided by $(\gamma - 1)$, which is a constant for any gas, and if θ is replaced by the absolute temperature of most of the gas, T then we get,

$$\frac{C^2}{cT(\gamma - 1)} = \frac{C^2}{\gamma RT} = \frac{C^2}{a^2} = (Ma)^2$$

(Where a is the speed of sound in the gas and Ma is the Mach Number)
Subsequently,

$$Nu = KF\{Pr, Re, (Ma)^2\}$$

The effect of the Mach number, Ma, on heat transfer can be ignored in most problems. However, for high-speed flow, large amounts of kinetic energy (i.e. velocity) are lost by friction in the wall layer near the surface and so the Mach number becomes an important variable [10],[14], [15] and [16].

4. Reynolds's Analogy

Reynolds assumes that the transfer of heat from a solid surface is similar to the transfer of momentum of a fluid from the surface, and therefore it is possible to express the transfer of heat in terms of frictional resistance to flow.
Consider a turbulent flow:
It can be assumed that particles of mass, m, transfer heat and momentum to and from the surface by moving perpendicular to the surface. Therefore, as an average, $q = \dot{m}c\theta$, is the heat transferred per unit area (where c = specific heat of the fluid, θ = temperature difference between the surface and most of the fluid).
Also, the rate of change in momentum through the flow is given by:

$$\dot{m}(C - C_w) = \dot{m}C$$

(Where C = velocity of most of the fluid; C_w = velocity of fluid at surface = 0)
Hence, force per unit area, $\tau_w = \dot{m}C$, (where τ_w is the shear stress in the fluid at the wall)
By unifying the equations for heat flow and momentum transfer, therefore,

$$\dot{m} = \frac{q}{c\theta} = \frac{\tau_w}{C}$$

$$or\ q = \frac{\tau_w c\theta}{C} \qquad (6)$$

In practice, for turbulent flow there is always a thin layer of fluid at the surface in which viscous effects prevail. This thin layer is known as the laminate sub-layer. In this layer, heat is transferred only by conduction.
Therefore, from Fourier's law for a unit area,

$$q = -k\left(\frac{d\theta}{dy}\right)_{y=0}$$

(Where k = thermal conductivity of the fluid; y = distance from the surface perpendicular to the surface).
Also, for viscous flow:

$$shear\ stress, \tau = \mu \times velocity\ gradient$$

Therefore, the shear stress at the wall is given by:

$$\tau_w = \mu\left(\frac{dC}{dy}\right)_{y=0}$$

(Where, μ = fluid viscosity; C = fluid velocity).
Now, since the laminated substrate is very thin it can be assumed that the temperature and velocity change linearly with distance from the wall, y,

$$i.e. q = -\frac{k\theta}{\delta_b} \text{‘} \tau_w = \frac{\mu C}{\delta_b}$$

(where $\delta_b$is the thickness of the laminated substrate)
By avoiding δ_b, and ignoring the negative sign, we get,

$$\frac{q}{k\theta} = \frac{\tau_w}{\mu C}$$

$$i.e. q = \frac{\tau_w k\theta}{\mu C}$$

It can be noted that this equation is identical to equation (6) when,
i.e. When $\frac{C\mu}{k} = 1$ or $Pr = 1$
Therefore, for fluids in which the Prandtl number (Pr) is approximately unity, Simple Reynolds's Analogy can be applied, since the heat transferred through the laminar substrate can be considered to have a similar behavior to the heat transferred from the sub-layer to most of the fluid. For most gases, dry steam, and superheated steam, the Prandtl number Pr lies between 0.65 and 1.2.
For unit surface area, $q = h\,\theta$, so by substituting into equation (6) we get,

$$\frac{h}{c} = \frac{\tau_w}{C}$$

Dividing both sides of the equation by ρC (where ρ is the average density of water) we get,

$$i.e. St = \frac{h}{\rho Cc} \quad (7)$$

The dimensionless friction factor, f is defined by:

$$f = \frac{\tau_w}{\left(\frac{\rho C^2}{2}\right)} \quad (8)$$

So, we have for Reynolds analogy,

$$St = \frac{f}{2} \quad (9)$$

Stanton's number, St, can be written as:

$$St = \frac{h}{\rho Cc} = \frac{hL}{k} \times \frac{\mu}{\rho CL} \times \frac{k}{c\mu} = \frac{Nu}{Re\,Pr}$$

$$i.e.\ St = \frac{Nu}{Re\,Pr} \quad (10)$$

The friction factor, f, can be derived mathematically for some cases, but in others a laboratory determination is necessary.
For turbulent flow in a pipe, a simple measurement of the pressure drop gives f, and thus the approximate heat flow can be found using Equation (6) or Equation (9);
For flow in a pipe with diameter d, the resistance to flow over a unit length is given by:

$$the\ resistance = \tau_w \pi d = \Delta P \frac{\pi}{4} d^2$$

(Where ΔP = pressure drop per unit length).

$$\therefore \tau_w = \frac{\Delta P d}{4} \quad (11)$$

An important factor in heat exchanger design is the pumping power required. Pumping power is the rate at which work is done to overcome frictional resistance.
i.e. For flow in a pipe;
Pumping capacity per unit length,

$$W = velocity \times \ frictional\ resistance$$

$$w = \tau_w \pi d C$$

$$heat\ flow\ per\ unit\ area, q = \frac{\tau_w c\theta}{C}$$

$$heat\ flow\ per\ unit\ length, Q = \frac{\tau_w c\theta\pi d}{C}$$

Therefore, the ratio of the pumping power, W, to the heat flow, Q, can be expressed as:

$$\frac{W}{Q} = \frac{\tau_w \pi d C^2}{\tau_w c\theta\pi d} = \frac{C^2}{c\theta} \quad (12)$$

For a heat exchanger, θ is the average logarithmic temperature difference.
It can be seen from Equation (12) that the power required for a given heat transfer rate can be reduced by reducing the flow velocity, C. However, reducing the fluid velocity means that the required surface area must be increased, and therefore a compromise must be made between the fluid velocity and the surface velocity.
Figure 2 below shows Simple Reynolds's Analogy.

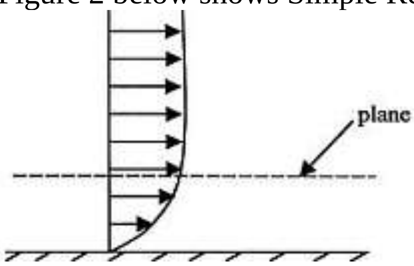

Figure 2: Simple Reynolds's Analogy

Various modifications have been made to the simple Reynolds analogy in an attempt to obtain an equation that will give a solution for turbulent heat transfer over a wide range of Prandtl numbers. (For a very viscous oil the Prandtl number is in the order of thousands, while for liquid minerals it is very low as 0.01). Equations based on modern theories of turbulent flow give the Stanton number as a function of the Reynolds number, the Prandtl number and the friction factor. In general, these equations can be reduced to $St = f/2$, when the Prandtl number is set equal to unity.
There are two additional points to mention here:
i. When the temperature difference between the surface and most of the fluid is very large, the property changes become large enough to be taken into account. Therefore, it is not long enough to use the mean film temperature to evaluate the properties, as given by equation (4). The change of each property with temperature through the flow must be known. Sometimes, with sufficient accuracy, an equation in the following form is used,

$$Nu = K\phi\left\{Pr, Re, \frac{T_S}{T_w}\right\}$$

(Where $T_S$and T_W are the absolute temperatures at the pipe axis and at the pipe wall, respectively, and the fluid properties are taken at the mean film temperature)
$h = \frac{1}{RA}$ ؛$R = \frac{1}{hA}$
ii. Equations for flow in a pipe usually do not take into consideration the effects of the entry length. At the entrance to a heated tube, hydro-dynamic and thermal boundary layers begin to form on the wall, and their thickness gradually increases until the flow becomes fully developed. In this initial region of the pipe, the heat transfer coefficient is much greater since the resistance to heat flow in the wall layer is less, and therefore an equation that ignores this effect will give a low value for the heat transfer that is calculated. This effect is more noticeable for laminar flow than turbulent flow, and is more important for fluids with high Prandtl numbers Pr. In most heat exchange procedures, the flow is turbulent and the length of the tube is long enough to make the effect of inlet length small enough to be ignored. In the case of oil coolers, the flow is laminar and the Prandtl number is high, so inlet effects can be noticeable. When flow through a flat plate is considered, the characteristic length dimension is taken as the distance from the leading edge. The heat transfer coefficient obtained is therefore the local value at that section of the plate. The average value of the heat transfer coefficient over the entire plate is the value used to calculate the heat transfer to or from the plate. It can be seen that the average heat transfer rate for a hot plate with a length L is twice the local heat transfer coefficient at a distance L from the leading edge [10], and [17] – [23].

5. Heat Exchanger Effectiveness

In certain cases of heat exchanger design, the efficiency of the heat transfer procedure becomes very important; As an example of compact heat exchangers, especially in aircraft manufacturing where weight is important, a Nusselt method later developed by Kays and London will be discussed in this section.
The effectiveness, $\in$ of a heat exchanger is defined as the ratio of the actual heat transferred to the maximum possible heat transferred.
For any heat exchanger with mass flow rates of hot and cold fluids $\dot{m}_H$ and $\dot{m}_C$ and specific heats c_H and c_C, let the total temperature changes for each fluid be Δt_H and Δt_C.
Ignoring losses to the surrounding environment,

$$Q = \dot{m}_H c_H \Delta t_H = \dot{m}_C c_C \Delta t_C$$
$$or\ Q = C_H \Delta t_H = C_C \Delta t_C \quad (13)$$

(Where $C_H = \dot{m}_H c_H$ and $C_C = \dot{m}_C c_C$ are the thermal capacities of hot and cold fluids).
From equation (13), it can be seen that a fluid with a smaller capacity, C, has a larger temperature change, Δt. The maximum possible temperature change for a fluid is $(t_{Hmax} - t_{Cmin})$, and this ideal temperature change can only be achieved with a fluid with a low heat capacity.

$$i.e.\ \ effectiveness, \epsilon = \frac{Q}{C_{min}\left(t_{H_{max}} - t_{C_{min}}\right)} = \frac{actual\ heat\ transferred}{maximum\ possible\ heat\ transferred} \quad (14)$$

The goal of a good heat exchanger design is to obtain the maximum possible fluid temperature change for a given driving force, that is, for a log mean temperature difference, LMTD. Therefore, a useful measure of heat exchanger efficiency is the number of heat transfer units, NTU, which is defined as:

$$NTU = \frac{(\Delta t)_{max}}{LMTD}$$

Where: $(\Delta t)_{max}$ is the maximum temperature change of one of the fluids.
now,
$Q = UA\ LMTD = C_{min}\ (\Delta t)_{max}$
$\therefore NTU = \frac{(\Delta t)_{max}}{LMTD} = \frac{UA}{C_{min}}$ (15)
The greater the number of heat transfer units, the greater the effectiveness of the heat exchanger.
The ratio of minimum to maximum heat capacity is usually given by the symbol R,

$$R = C_{min}/C_{max} \quad (16)$$

Note that R can vary between 1 (when both fluids have the same heat capacity) and 0 (when one fluid has infinite thermal capacity, e.g. condensed vapor or boiling liquid).

Figure 3 below shows a typical example of an effectiveness plot, $\in$, against NTU for varying values of the heat capacity ratio, R.

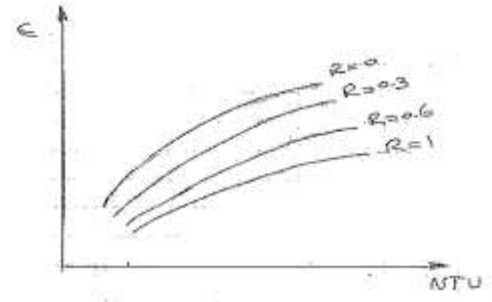

Figure 3: Plot of Effectiveness against the Number of Heat Transfer Units

Consider a counter flow Heat exchanger as shown in Figures 4 and 5 below.

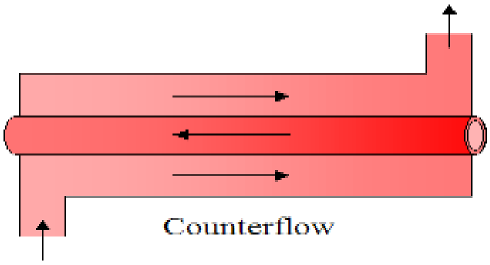

Figure 4: Counter Flow Heat Exchanger

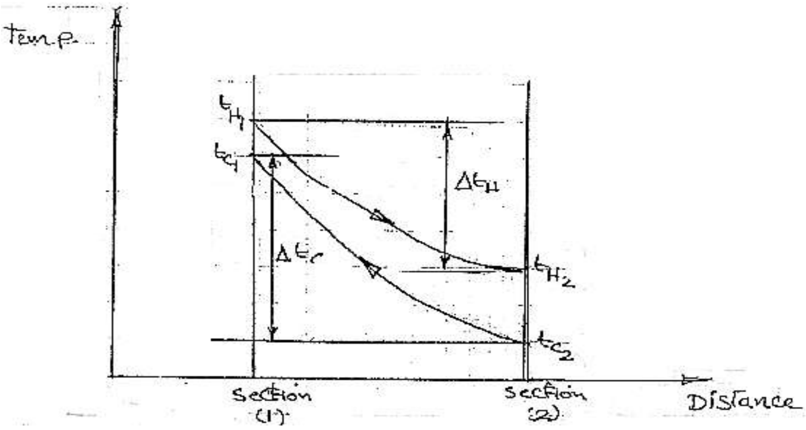

Figure 5: Temperature versus Distance Plot for a Counter – Flow Heat Exchanger

From Figure 5, it can be seen that, $C_c = C_{min}$ since $\Delta t_C > \Delta t_H$

$$R = C_{min}/C_{max} = C_C/C_H$$

Or using equation (13),

$$\frac{C_C}{C_H} = \frac{\Delta t_H}{\Delta t_C}$$

The following equation could be obtained,

$$R = \frac{t_{H_1} - t_{H_2}}{t_{C_1} - t_{C_2}} \quad (17)$$

From equation (14),

$$\in = \frac{Q}{C\left(tC_{min_{H_{max}}}\right)\dfrac{C\left(t_{C_1} - t_{C_2}\right)_{min}}{C\left(t_{H_1} - t_{C_2}\right)_{min} = \dfrac{t_{C_1} - t_{C_2}}{t_{H_1} - t_{C_2 min}}}}$$

$$effectiveness, \epsilon = \frac{Q}{C_{min}\left(t_{H_{max}} - t_{C_{min}}\right)} = \frac{C_{min}(t_{C_1} - t_{C_2})}{C_{min}(t_{H_1} - t_{C_2})} = \frac{(t_{C_1} - t_{C_2})}{(t_{H_1} - t_{C_2})} \quad (18)$$

From equation (15),

$NTU = \frac{UA}{C_{min}} = \frac{t_{C_1} - t_{C_2}}{LMTD}$

From the equation of logarithmic mean temperature difference shown below,

$$LMTD = \frac{\left(t_{H_1} - t_{C_1}\right) - \left(t_{H_2} - t_{C_2}\right)}{log_e \frac{\left(t_{H_1} - t_{C_1}\right)}{\left(t_{H_2} - t_{C_2}\right)}}$$

$$\therefore NTU = \frac{\left(t_{C_1} - t_{C_2}\right)}{\left(t_{H_1} - t_{C_1}\right) - \left(t_{H_2} - t_{C_2}\right)} log_e \frac{\left(t_{H_1} - t_{C_1}\right)}{\left(t_{H_2} - t_{C_2}\right)}$$

$$or\ NTU = \frac{\left(t_{H_1} - t_{H_2}\right)\left(t_{C_1} - t_{C_2}\right)}{\left(t_{C_1} - t_{C_2}\right)} = log_e \left\{ \frac{\left(t_{H_1} - t_{C_2}\right) - \left(t_{C_1} - t_{C_2}\right)}{\left(t_{H_1} - t_{C_2}\right) - \left(t_{H_1} - t_{H_2}\right)} \right\}$$

$$\therefore NTU(R-1) = log_e \left\{ \frac{\left[\left(t_{C_1} - t_{C_2}\right)/\in\right] - \left(t_{C_1} - t_{C_2}\right)}{\left[\left(t_{C_1} - t_{C_2}\right)/\in\right] - R\left(t_{C_1} - t_{C_2}\right)} \right\}$$

Using equations (17) and (18),

$$NTU(R-1) = log_e \frac{(1-\in)}{(1 - R \in)}$$

$$or\ \in = \frac{1 - e^{-NTU(1-R)}}{1 - Re^{-NTU(1-R)}} \quad (19)$$

For a counterflow heat exchanger when $C_H = C_C$, i.e. R = 1 (say a gas turbine heat exchanger), so the expression for efficiency cannot be obtained by substituting R = 1 into equation (19). For this case, the temperature change for each fluid is the same, since $C_H = C_C$, so the LMTD is equal to the temperature difference between the hot and cold fluids, which remains constant throughout the heat exchanger. Therefore, the equation is written as $NTU = \frac{(t_{C_1} - t_{C_2})}{(t_{H_1} - t_{H_2})}$ and thus the derivation follows as above, giving [10], and [24] – [28]:

$\in = \frac{NTU}{1+NTU} \quad (20)$

For a parallel-flow heat exchanger, it can be explained that:

$$\in = \frac{1 - e^{-NTU(1+R)}}{1 + R} \quad (21)$$

When R = 0, i.e. In the case of a condenser, it can be observed from Equation (19) or Equation (21) that the effectiveness is,

$$\in = 1 - e^{-NTU} \quad (22)$$

6. Natural Convection

As mentioned previously, heat transfer by free or natural convection is the result of differences in density with respect to the fluid, causing a natural circulation, and thus heat transfer. For most problems in which a fluid flows across a surface, the superimpose effect of natural convection is small enough to be ignored. When there is no forced fluid velocity, heat is transferred entirely by natural convection (when radiation is ignored). Transmission in this case depends on the coefficient of cubical expansion, β, which is given by:

$$\rho_1 = \rho_2(1 + \beta\theta)\ or\ (\rho_1 - \rho_2) = \rho_2\beta\theta$$

(Where θ is the temperature difference between the two parts of the fluid with densities ρ_1 and ρ_2). The upward compression force per unit volume of fluid is up thrust per unit volume, and the speed of the load current is dependent on the upward compression. Natural convection depends on the heat transfer coefficient, which in turn depends on the viscosity of the fluid and the thermal conductivity of the material, over a distinct length.

Since the cubic expansion coefficient β and the local acceleration due to gravitation g do not have a separate effect on heat transfer, only their product βg should be considered.
For dimensional analysis we get,

$$h = A\mu^{a_1}\rho^{b_1}k^{c_1}c^{d_1}\theta^{e_1}(\beta g)^{f_1}L^{g_1} + B\mu^{a_2}\rho^{b_2}k^{c_2}c^{d_2}\theta^{e_2}(\beta g)^{f_2}L^{g_2} + etc$$

Therefore, by the same steps as in forced convection it can be shown that:

$$Nu = KF\left[\frac{c\mu}{k}, \frac{\beta g\rho^2L^3\theta}{\mu^2}\right]$$

$$or \quad Nu = KF\{Pr, Gr\}$$

Where: $Gr = \frac{\beta g\rho^2L^3\theta}{\mu^2} = \frac{\beta gL^3\theta}{\nu^2}$ is called the Grashof Number.
In many cases of natural convection, it is possible to use an approximate equation to evaluate the heat transfer coefficient h.
For example, for a natural convection from a horizontal pipe,

$$when\ 10^4 < Gr < 10^9, \qquad h = 1.32\left(\frac{\theta}{d}\right)^{1/4}$$

$$when\ 10^9 < Gr < 10^{12}, \qquad h = 1.25\theta^{1/3}$$

(Where $h\ is\ in\ W/m^2K, \theta\ is\ in\ K, d\ is\ in\ m$).
Figure 6 below shows heat transfer by natural convection.

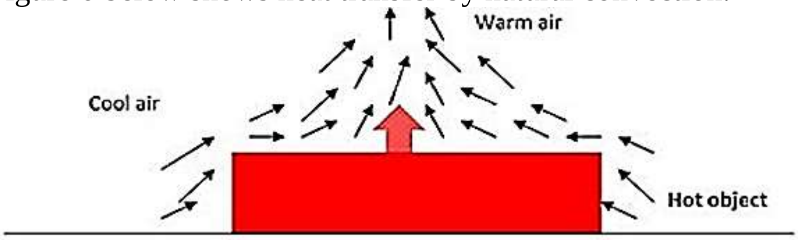

Figure 6: Heat Transfer by Natural Convection

For a natural convection from a vertical wall, the air, as it rises as a result of convection currents, forms a wall layer, starting from the bottom and gradually thickening at the top of the wall. The figure below (Figure 7) shows the behavior of wall layer formation on a vertical wall.
The heat transfer coefficient changes at the top of the wall. The formulas for heat transfer from a vertical wall give a local heat transfer coefficient at a distance, L, from the bottom of the wall, where the characteristic linear dimension used in the Grashof number is the length, L.
It can be clarified that the average value of the heat transfer coefficient from the bottom upwards for the distance L is given by:

$$h_{av} = \frac{3}{4}h$$

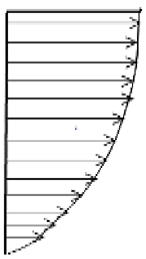

Figure 7: The Behavior of Wall Layer Formation on a Vertical Wall
(Where h_{av} is the average heat transfer coefficient, and h is the heat transfer coefficient at the section that is L away from the bottom of the wall) [10], [29] and [30].

7. Conclusion

Convection refers to the method of energy transfer between a solid surface and the nearby liquid or gas that is in motion. This process involves both conduction and fluid movement. The rate of convective heat transfer increases with the speed of the fluid. When there is no significant fluid movement, heat is transferred solely through conduction. However, when the fluid is in motion, it enhances the heat transfer between the solid surface and the fluid, adding complexity to the assessment of heat transfer rates.

The convective heat transfer process plays a crucial role in numerous practical applications, including porous insulations, the cooling of rotating electric windings, geothermal reservoirs, irrigation systems, heat exchangers, and the exploration of oil and gas fields.

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