

Overview and Mathematical Analysis of Heat Transfer in Boiling

Osama Mohammed Elmardi Suleiman Khayal¹, Fathelrahman Ahmed Elmahi², Awadalla Ahmed³
and Ahmed Ali Mustafa Mohammed⁴

1,2,3,4Department of Mechanical Engineering, Faculty of Engineering and Technology, Nile Valley University, Atbara – Sudan and Elsheikh Abdallah Elbadri University, Berber – Sudan

Email: osamamm64@gmail.com

Abstract

Boiling heat transfer plays a vital role across numerous industries, facilitating effective heat transfer via the liquid-to-vapor phase change. Its significance is especially pronounced in power generation and chemical processing, where it impacts both design and safety considerations. Gaining a comprehensive understanding of this process can enhance energy efficiency in heating and cooling systems. This phenomenon is essential not only for large power plants but also for small electronic devices, enabling engineers to optimize thermal energy management and foster innovation. Improving boiling heat transfer in power facilities can lead to substantially greater steam generation efficiency, resulting in increased electricity production.

Keywords: Pool Boiling; Forced Convection Boiling; Sub – Cooled or Local Boiling; Boiling Regimes; Interface Evaporation; Nucleate Boiling.

1. Introduction

In the past, research on convective heat transfer primarily focused on homogeneous systems with a single phase. However, certain convection processes involve phase changes, such as boiling and condensation. Boiling refers to the transition from a liquid phase to a vapor phase, whereas condensation is the process of changing from vapor phase back to liquid phase.

Heat transfer during phase changes, including boiling and condensation, has numerous practical applications, such as:

- i. Cooling systems for nuclear reactors and rocket motors.
- ii. Steam power plants, including boilers and condensers.
- iii. Refrigeration and air conditioning systems, featuring evaporators and condensers.
- iv. Melting metals in furnaces.
- v. Operations in refineries and sugar mills, such as heat exchangers.
- vi. Process heating and cooling.

Figure 1 below illustrates the boiling patterns associated with heat transfer through boiling [1] – [15].

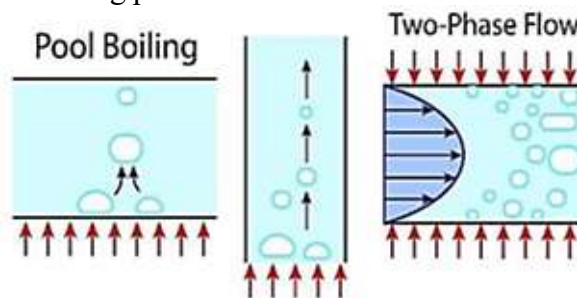


Figure 1: The Boiling Patterns in Heat Transfer by Boiling

2. Overview of Boiling and Condensation

The processes of boiling and condensation exhibit several distinctive characteristics [1], [3], [6], and [10]:

- i. During these phase transitions, heat can be transferred to or from the fluid without altering its temperature.
- ii. The heat transfer coefficients and rates associated with latent heat during phase changes are typically higher than those observed in normal convection processes (i.e., processes without phase change).
- iii. A significant rate of heat transfer can be achieved with only a small temperature difference.

3. Complexities Involved in Boiling and Condensation

The phenomena linked to boiling and condensation are inherently more intricate than those of normal convection, due to the following factors [1], [3], [6], and [10]:

- i. The effects of latent heat.
- ii. The influence of surface tension.
- iii. The characteristics of the surface and other attributes of two-phase systems.

4. Heat Transfer During Boiling

4.1 General Characteristics

Boiling is a convective heat transfer process that involves a phase transition from a liquid to a vapor state. It can also be described as the evaporation phenomenon occurring at the surface of a liquid. This process requires that the surface temperature (t_s) exceeds the saturation temperature corresponding to the fluid pressure (t_{sat}). Heat is transferred from a solid surface to a liquid based on the following principle:

$$Q = hA_s(t_s - t_{sat}) = hA_s\Delta t_e$$

Where, $\Delta t_e = (t_s - t_{sat})$ which is known as the excess temperature.

The boiling process has several applications, detailed in the following instances [1], [3], [6], and [10]:

- i. **Steam Generation:** Employed in power generation, various industrial applications, and vacuum heating within both steam and nuclear power facilities.

- ii. **Heat Absorption:** Utilized in refrigeration and air conditioning systems for effective temperature regulation.
- iii. **Distillation and Refining:** Essential for separating and purifying substances in chemical processes.
- iv. **Concentration and Dehydration:** Important for the concentration, drying, and dehydration of food products and other materials.
- v. **Cooling Systems:** Applied in machines like nuclear reactors and rocket engines, where substantial heat must be dissipated in a compact volume, with heat loss rates reaching up to 10^8 W/m^2 and heat transfer rates in modern boilers typically around $2 \times 10^5 \text{ W/m}^2$.

4.2 Uses of the Boiling Process

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4.3 Types of Heat Transfer Through Boiling

Heat transfer through boiling can manifest in several forms [1], [3], [6], and [10] as stated in paragraphs below:

4.3.1 Pool Boiling

In pool boiling, the liquid resting above the heated surface remains largely stationary, with any movement near the surface resulting from natural convection and the mixing caused by the formation and release of bubbles. This type of boiling is found in steam boilers that utilize natural convection. Figure 2 below illustrates the pool boiling curve.

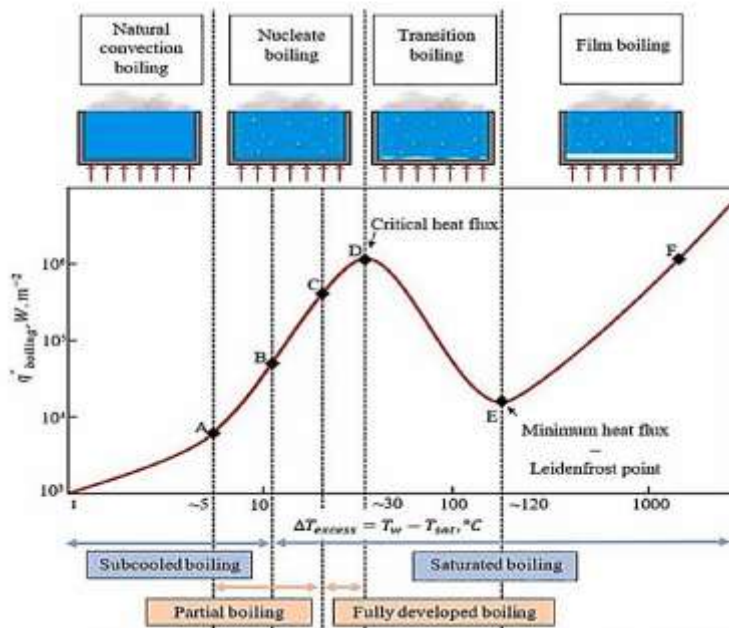


Figure 2: The Pool Boiling Curve

4.3.2 Forced Convection Boiling

In this scenario, the fluid movement is driven by external factors, including natural convection and the mixing caused by induced bubbles. The fluid is actively pumped to promote flow. This type of boiling occurs in water tube boilers where forced convection is present. Figure 3 below illustrates the boiling curve associated with forced convection.

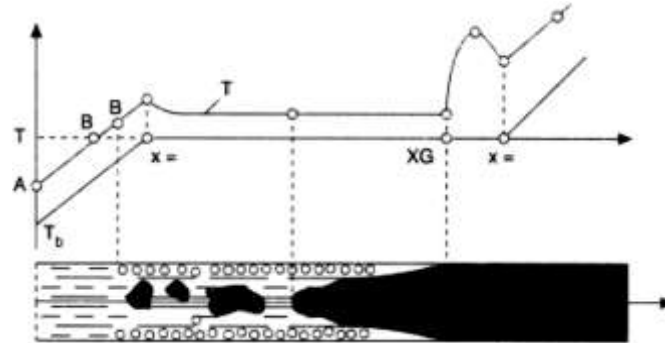


Figure 3: The Forced Convection Boiling Curve

4.3.3 Sub – Cooled or Local Boiling

In this scenario, the liquid's temperature is lower than its saturation temperature, causing bubbles to form near the heat surface. These bubbles quickly condense after passing through the cooler liquid, which remains below the boiling point. Figure 4 below illustrates the cooling curve at the sub-cooled or local boiling point.

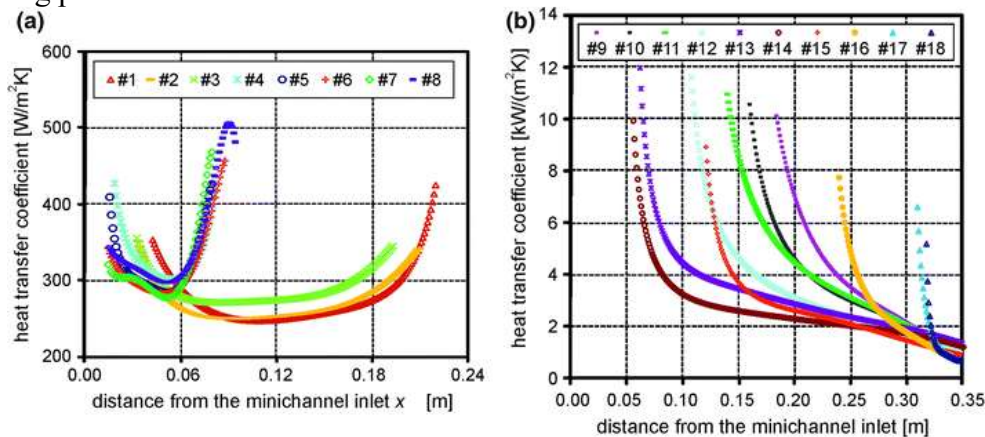


Figure 4: The Cooling Curve in the Sub – Cooled Point or Local Boiling Point

4.3.4 Saturated Boiling

In this scenario, the temperature of the liquid is higher than the saturation temperature. Vapor bubbles that form at the solid surface, also known as the liquid-solid interface, are propelled through the liquid by buoyancy and rapidly release into the atmosphere at the free surface (liquid-vapor interface). The mechanism of saturated boiling is illustrated in Figure 5 below.

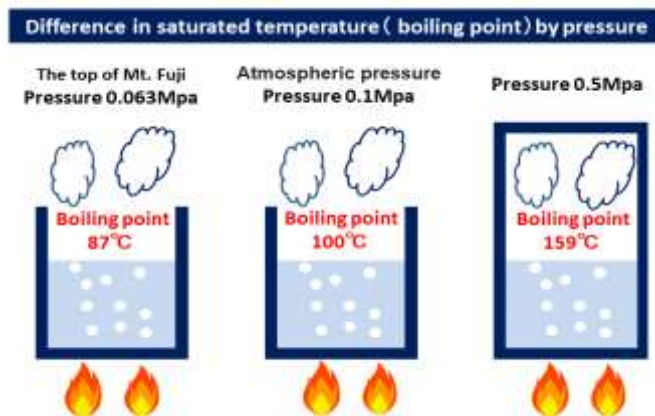


Figure 5: The Mechanism of Saturated Boiling

4.4 Boiling Zones or Boiling Systems (Boiling Regimes)

The process of boiling is influenced by the characteristics of the surface, the thermo-physical properties of the liquid, and the behavior of vapor bubbles. Due to the complexity and multitude of factors involved, there are no universal equations that can adequately describe the boiling process. Nevertheless, substantial advancements have been made in understanding the physical mechanisms behind boiling [1], [3], [6], and [10].

As illustrated in Figure 6, the temperature distribution within a saturated boiling basin at the liquid-vapor interface is depicted. The figure demonstrates that, despite a significant decrease in temperature near the solid surface, the temperature in the majority of the liquid remains just above the saturation point. Consequently, bubbles formed at the solid-liquid interface ascend and move through the liquid-vapor interface. When boiling occurs due to pool boiling or forced convection, three distinct boiling systems are identified (interface evaporation, nucleate boiling, and film boiling), which correspond to an increasing heat flux, as shown in Figure 6. This specific curve was derived from experiments using an electrically heated platinum wire placed in a basin of water (at its saturation temperature), where varying the wire's surface temperature allowed for the measurement of the surface heat flux (q_s).

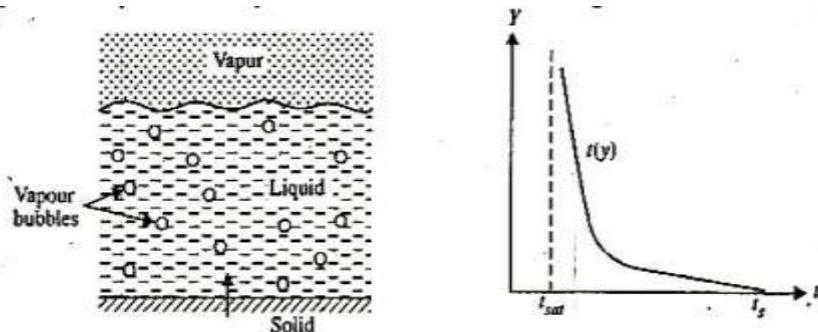


Figure 6: Pool Boiling at a Liquid – Vapor Interface

4.4.1 Interface Evaporation

Interface evaporation, which occurs without bubble formation, takes place in Region I, also referred to as the free convection zone. In this area, the excess temperature, Δt_e , is minimal, measuring only 5°C. The liquid close to the surface experiences slight superheating, while convection currents circulate the liquid, facilitating evaporation at the surface. Below, Figure 7 illustrates the boiling curve for water.

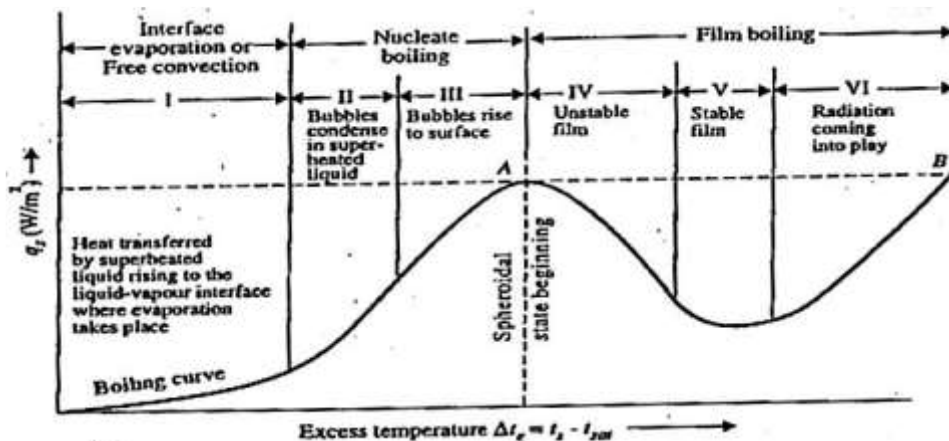


Figure 7: The Boiling Curve for Water

4.4.2 Nucleate Boiling

Nucleate boiling occurs in regions II and III as illustrated in Figure 7. As the temperature difference (Δt_e) (over-temperature) increases, bubbles start to form on specific local points on the surface of the wire. These bubbles condense in the liquid before they can reach the surface. This phenomenon characterizes region II, where nucleate boiling initiates. With a further rise in (Δt_e), bubble formation accelerates, and the bubbles ascend to the liquid's surface, resulting in rapid evaporation, as depicted

in region III. Nucleate boiling is defined by the emergence of bubbles at nucleation sites, which subsequently induces agitation in the liquid. The movement of these bubbles facilitates the mixing of a substantial volume of fluid, leading to a marked increase in heat flux and the boiling heat transfer coefficient. (It is essential that the boiling equipment is specifically designed to function within this range.)

Nucleate boiling can be observed up to an (Δt_e) value of 50°C. The peak heat flux, referred to as the critical heat flux, is reached at point A (see Figure 7), with a value of 1 MW/m².

4.4.3 Film Boiling

The film or slice boiler encompasses regions IV, V, and VI, as illustrated in Figure 7 above. In region IV, known as the strip boiling region, the trend of increasing heat flux with rising overheating (observed up to region III) is reversed. This shift occurs because bubbles form rapidly, covering the heating surface and preventing fresh liquid from coming into contact with it. These bubbles quickly coalesce to create a vapor film that blankets the entire surface. As the thermal conductivity of the vapor layer is lower than that of the liquid, the heat flux decreases by approximately (Δt_e).

Within the temperature range of 50°C < Δt_e < 150°C, the boiling states fluctuate between nucleate boiling and laminar boiling, leading to a phase referred to as transition boiling, unstable laminar boiling, or partial laminar boiling (region IV). As (Δt_e) continues to rise, the vapor film stabilizes, thoroughly enveloping the heating surface and resulting in a minimal heat flux, as depicted in region V. The surface temperatures required to maintain a stable vapor layer are significantly high, and under these conditions, considerable heat is lost due to radiation, as indicated in region VI. The phenomenon of stable film boiling can be observed when a droplet of water lands on a glowing hot surface. Instead of evaporating immediately, the droplet dances lightly on the surface, which is attributed to the formation of a stable vapor film at the interface between the heated surface and the liquid droplet.

4.4.4 Critical Heat Flux or Burnout Point

The critical heat flux, often referred to as the burnout point (indicated as point A in Figure 7 above), represents the maximum heat flux on the boiling curve where the transition from nucleate boiling to film boiling occurs. This point is also known as the boiling crisis, as the boiling process beyond this stage becomes unstable unless point B is achieved. The temperature at point B is significantly high, typically exceeding the melting point of the solid material. If the heating of the metal surface surpasses point B, there is a risk that the metal may degrade or even melt. Consequently, point A is frequently termed the boiling crisis or combustion point.

4.4.5 Considerations for Bubble Shape and Size

The heat transfer rate during nucleate boiling is significantly affected by the characteristics of the heating surface and the surface tension at the solid-liquid interface. In contrast, the shape, size, or inclination angle of the bubbles has a minimal impact on heat transfer rates. Surface tension pertains to how well the liquid wets the surface (where lower surface tension indicates a higher wetting ability), influencing the contact angle between the bubbles and the solid surface. Surface contamination can alter these wetting properties, subsequently affecting the size and shape of the vapor bubbles.

When the liquid exhibits low surface tension, it tends to effectively wet the surface, allowing bubbles to be easily pushed by the liquid and rise. The liquid shearing off the bubbles leads to a transformation in their shape, often resulting in a globular or oval form, as depicted in Figure 8(i) for a fully wetted surface. In cases where the liquid has intermediate surface tension (resulting in a partially wetted surface), a temporary equilibrium can exist between the bubbles and the solid surface, necessitating the formation of larger bubbles before they can be released by buoyancy. The corresponding bubble shape in this scenario is illustrated in Figure 8(ii).

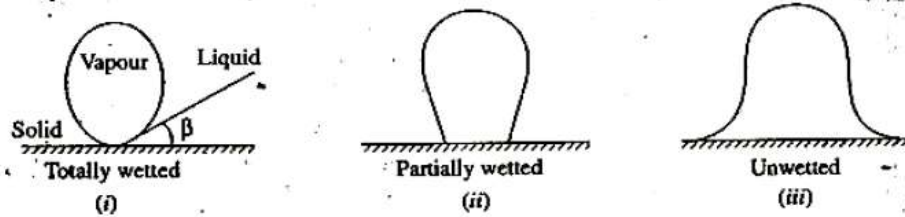


Figure 8: Typical Shapes of Vapor Bubbles

On the dry surface (Figure 8(iii)), bubbles spread out and create a wedge between the water and the heating surface. This configuration enables hydrostatic forces to counterbalance the buoyancy effect. As illustrated in Figure 8(i), bubble formation leads to a higher heat transfer rate compared to the bubble shapes depicted in Figures 8(ii) and 8(iii). Research has shown that incorporating certain materials to lower surface tension has a similar impact as applying a moisturizing surface, resulting in enhanced heat transfer rates.

4.4.6 Bubble Growth and Collapse

Experimental observations indicate that bubbles do not consistently maintain thermodynamic equilibrium with the surrounding liquid. The temperature of the vapor within the bubble may not align with that of the liquid. To analyze this, let's examine the forces acting on a spherical vapor bubble, as illustrated in Figure 9. The compressive forces exerted on the bubble must be counterbalanced by the surface tension at the interface between the vapor and liquid. Therefore,

$$\pi r^2(p_v - p_l) = 2\pi r \cdot \sigma \quad (1)$$

$$\text{or } p_v - p_l = \frac{2\sigma}{r} \quad (2)$$

Where:

p_v = vapor pressure inside the bubble

p_l = pressure of the liquid above the bubble surface

σ = surface tension of the vapor-liquid interface

Steam can be considered as an ideal gas where the Clay Peron equation can be used which is given below:

$$\frac{dp}{p} = \frac{h_{fg}}{RT^2} dT \quad (3)$$

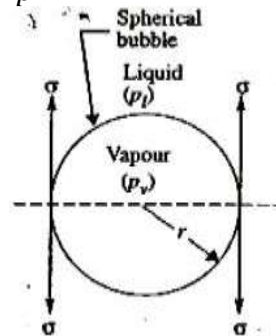


Figure 9: Balance of Forces on a Spherical Vapor Bubble

Where: h_{fg} = latent heat of evaporation.

From the ideal gas law:

$$\frac{P}{RT} = \rho_v$$

(Where R = gas or vapor constant; ρ_v = density of the vapor formed)

Substituting the above equation into equation (3) and rearranging, we get:

$$\frac{dp}{dT} = \frac{h_{fg} \cdot \rho_v}{T}$$

$$\text{or } \frac{p_v - p_l}{T_v - T_{sat}} = \frac{h_{fg} \cdot \rho_v}{T_{sat}} = \frac{p \cdot h_{fg}}{RT_{sat}} \quad (4)$$

Where:

T_v The temperature of the vapor inside the bubble.

T_{sat} The saturation temperature of the vapor inside the bubble at p_v .

From equations (2) and (4) we get:

$$T_v - T_{sat} = \frac{2\sigma}{r} \left[\frac{R}{P} \cdot \frac{T_{sat}^2}{h_{fg}} \right] \quad (5)$$

The above equation suggests the following: If $(T_l - T_{sat}) > (T_v - T_{sat})$, then a bubble with radius r will grow or explode. T_l is the temperature surrounding the bubble.

Referring to Figure 10, the maximum diameter of the bubble formed on the heating surface depends on the following variables:

σ_{lv} = the tension between liquid and vapor.

σ_{ls} = the tension between the liquid and the solid surface.

σ_{vs} = the tension between the vapor and the solid surface.

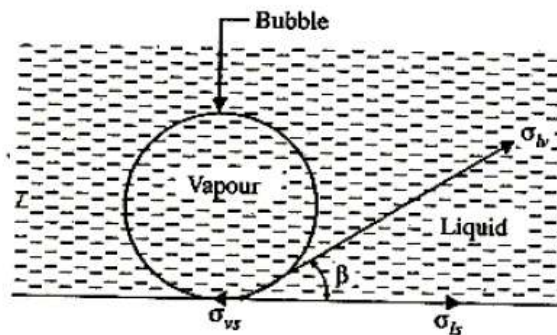


Figure 10: Critical Diameter of a Bubble

β = the angle formed by the bubble as shown in Figure (10.5).

ϕ_c = maximum or critical diameter of the bubble.

$g(p_l - p_v)$ = buoyancy force.

thus,

$$d_c = \int \left[\beta, \sigma_{lv}, g(\rho_l - \rho_v), \frac{\sigma_{lv}}{\sigma_{ls}} \right]$$

Using dimensional analysis technique, we obtain:

$$d_c = C \cdot \beta \left[\frac{\sigma_{lv}}{\sigma_{ls}} \right] \sqrt{\frac{\sigma_{lv}}{g(\rho_l - \rho_v)}} \quad (6)$$

Where C is a constant generally calculated with laboratory results.

C value=0.0148 for water bubble.

4.4.8 Factors Affecting Nucleate Boiling

Nucleation boiling is affected by the following factors:

I. Shape of the Material and Condition of the Heating Surface

The boiling heat transfer coefficient depends greatly on the material of the heating surface. Under identical conditions of pressure and temperature difference, it is different for different metals (for example copper has a higher value than steel, zinc and chromium).

Heat transfer rates are also affected by the condition of the heating surface. A rough surface gives better heat transfer than if the surface is smooth or painted (smoothness weakens the tendency of the metal to wetting or hydrate).

The shape of the heating surface also affects heat transfer.

II. Properties of Liquids

Experiments show that as the dynamic viscosity of a liquid increases, the size of the bubbles also grows. Larger bubble sizes lead to a decrease in the frequency of bubble formation, which in turn reduces heat transfer efficiency. Furthermore, a liquid with high thermal conductivity enhances the rate of heat transfer.

III. Pressure

Pressure influences the rate at which bubbles grow, which in turn impacts the temperature difference ($t_s - t_{\infty}$) that drives heat transfer. In the case of a boiling liquid, the maximum permissible heat flux initially increases with pressure until it reaches a critical level, after which it begins to decrease.

IV. Mechanical Agitation

Experiments demonstrated that as the intensity of stirring increases, the rate of heat transfer also rises.

4.4.9 Boiling Correlation

In boiling heat transfer, the driving force is the excess temperature, which is given by the equation:

$$\Delta t_e = t_s - t_{sat} \quad (7)$$

The governing equation for the boiling process is,

$$Q = hA \Delta t_e$$

Where h is the boiling point coefficient.

Since there is no analytical solution available for heat transfer by boiling due to the difficult behavior of the fluid, empirical equations or relationships are used for engineering calculations, some of which are given in the following side headings:

4.4.9.1 Nucleate Pool Boiling

I. For nucleation pool boiling Rosenhow recommends the following cross-correlation:

$$q_s = \mu_l \cdot h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{0.5} \left[\frac{C_{PL} \cdot \Delta t_e}{C_{SL} \cdot h_{fg} \cdot pr_l^n} \right]^3 \quad (8)$$

Where:

q_s = surface heat flux w/m^2

μ_l = viscosity of liquid kg/ms

h_{fg} = enthalpy of vaporization J/kg

ρ_l = density of saturated liquid kg/m^3

ρ_v = density of saturated steam kg/m^3

σ = surface tension of the liquid-vapor interface, N/m

C_{PL} = specific heat of saturated liquid, J/kgk

$\Delta t_e = (t_s - t_{sat})$ = excess temperature, k

C_{SL} = surface fluid constant (determined from laboratory data)

n = another constant that depends on the liquid and the surface; For water $n=1$, while for other liquids $n=1.7$.

The value of C_{SL} is given in Table 1 below:

Table 1: Evaluation of C_{SL} for Pool Boiling

S. No.	Liquid – surface	C_{SL}
1	Water – copper	0.013
2	Water – brass	0.060
3	Water – platinum	0.013
4	Water – ground and polished stainless steel	0.008
5	Water – mechanically polished stainless steel	0.013
6	Benzene – chromium	0.010
7	Ethanol – chromium	0.0027
8	n-pentane – chromium	0.0150
8	n-pentane – copper	0.003
10	Isopropyl alcohol – copper	0.00225

II. Jacob proposed the following correlation for nucleation boiling at atmospheric pressure on a flat plate and with low heat flux.

$$Nu = 0.16(Gr.Pr)^{0.33} \quad (9)$$

iii/ For nucleation boiling on a vertical plane plate, the Jacob cross-correlation is of the form:

$$Nu = 0.61(Gr.Pr)^{0.25} \quad (10)$$

4.4.9.2 Critical Heat Flux for Nucleate Pool Boiling

On the boiling curve, the critical heat flux is an important point. It is always desirable to run the boiling process close to this point. In 1958, Zuber proposed the following expression for such a case:

$$q_{sc} = 0.18(\rho_v)^{1/2} h_{fg} [g\sigma(\rho_l - \rho_v)]^{1/4} \quad (11)$$

The given expression is independent of the fluid's viscosity, conductivity, and specific heat.

4.4.9.3 Film Pool Boiling

In steady-state boiling, heat transfer arises from both convection and radiation. In 1950, Bromley proposed the following cross-correlation for film boiling from the outer surface of horizontal tubes:

$$(h)^{4/3} = (h_{conv.})^{4/3} + h_{rad.} (h)^{1/3} \quad (12)$$

Equation (12) is tiring and cumbersome to solve, so it can be written within an error of $\pm 5\%$ as follows:

$$h = h_{conv.} + \frac{3}{4} h_{rad} \quad (13)$$

The convective coefficient, $h_{conv.}$, is given in the absence of radiation as follows:

$$h_{conv.} = 0.62 \left[\frac{k_v^3 \rho_v (\rho_l - \rho_v) g (h_{fg} + 0.4 C_{pv} \Delta t_e)}{\mu_v D \Delta T_e} \right]^{1/4} \quad (14)$$

Where D is the outer diameter of the pipe. The vapor properties in the above equation are evaluated at the arithmetic mean surface and saturation temperatures.

Radiant heat transfer coefficient,

$$h_{rad} = \frac{5.67 \times 10^{-8} \epsilon (T_s^4 - T_{sat}^4)}{(T_s - T_{sat})} \quad (15)$$

Where ϵ is the emissivity of the solid.

5. Conclusion

Heat transfer processes can modify temperatures and induce phase changes, such as melting and boiling, which are optimized in engineering applications. Boiling is widely used in both residential and industrial contexts, including refrigeration, power generation, and cooking. It also purifies liquids by eliminating impurities, making it a common method for sterilizing water and preparing beverages like tea and coffee. Despite its common applications, boiling remains a subject of active research in chemistry and physics, with scientists studying its dynamics and the effects of various factors like pressure and temperature. This research has important implications for fields such as energy production and materials science.

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